INTERNATIONAL JOURNAL OF HIGHER EDUCATION AND RESEARCH
IJHER, Vol., 10 (2), 2020, 254-271.
www.ijher.com

IJHER
(ISSN 2277 260X)

CODEN:IJHER ORIGINAL ARTICLE

A DETERIORATION OF INVENTORY MODEL FOR EXPONENTIAL DEMAND AND CONSTANT HOLDING COST, DETERIORATION COST WITH DIFFERENTIAL EQUATION AND CUBIC EQUATION

Namrata Tripathi
Assistant Professor, Department of Mathematics
Govt, P.G. College, Rajgarh (M.P), India
Author's Email: ntripathi16@gmail.com

ABSTRACT:
*The goal of this research paper most deteriorated item of inventory item, the demand rate has been considered a constant function. But in a real life these holding cost, deterioration cost vary over time. We develop a deterministic deterioration inventory item with effect in demand rate is exponential. The mathematical model is solved numerically by minimizing the total inventory cost per cycle time. The next target of this research paper is to develop a mathematical model for deteriorating items such as e.g vegetables, milk, meat, radioactive materials, volatile liquids, etc with exponential demand using third order equation to determine optimal solution. (**2010 Mathematics subject classification:** 90B05)*
Keywords: *Cubic equation, Inventory item, Deterioration, Exponential growth and Cycle* time.

[Tripathi, N. A Deterioration Of Inventory Model For Exponential Demand And Constant Holding Cost, Deterioration Cost With Differential Equation And Cubic Equation. *International Journal of Higher Education and Research* 2020;10(2):254-271]. ISSN 2277 260X (online). http://www.ijher.com. doi:10.7755/ijher170720.16

INTRODUCTION

In this research paper a model of the inventory item of the economic order quantity for the deterioration of the items in which there is no consent and the value of the recovery is due to the loss of deterioration items. In this mathematical model demand rate is seen as exponential function. This mathematical model resolved analytically minimizing the total inventory time . A numerical example and analysis of the sensitivity of the model is demonstrated by proposed mathematical model. (Ballou, 2007) and (Dave and Patel, 1981) considered deteriorating to be a constant fraction of the on hand inventory while the demand rates is changing linearly. Moreover the cost of material handling plays a significant role in increasing the overall cost of production. To achieve the targets of production at the minimum cost a balance between demand and supply of material inventory is created for optimum utilization of resources. There is a need to effectively monitor and control the cycle of production without affected by the supply of material inventory by (Rastogi, 2010). The utility has proposed a model by which an optimum solution is derived as a new approach of third order linear equation for those items whose demand changes with time and constant deterioration rate. The traditional inventory models results in depletion of inventory which is caused by a constant demand rate Chung and (Tsai,2001). (Yong et al., 2010) discussed the production inventory model for deteriorating items to generate more selling opportunities in multiple market demands. (Singh, S, et al.,2012) developed an inventory model for decaying items with selling price dependent demand in inflationary environment. (Widyadana et. al.,2012) developed EPQ models for deteriorating items with preventive maintenance, random machine breakdown and immediate corrective action. Corrective and preventive maintenance times are assumed to be stochastic and the unfulfilled demands are lost sales. (Zeinab Sazvar et al.,2011) developed an inventory model for a main class of deteriorating items, under stochastic lead time assumption and a non-linear holding cost is considered. (Mishra et al.,2013) considered a deterministic inventory model with time-dependent demand and time-varying holding cost where deteriorating is time proportional and the model considered here allows shortages and the demand is partially backlogged.This paper provides an optimum solution using a sensitivity analysis for management of raw material and finished goods.

(Pahl et al.,,2014) globalization has created many challenges for the business world. The expansion of business involves management of production process which in turn requires proper management and control of inventory. Some items of the inventory are subjected to fast rate of deterioration. Hence there is needed to monitor an effective inventory planning and control. (Bouras et al., 2015) studied three level stock categories for production system boosted by the effective advertisement policy of a firm. The three level stock systems involve manufactured items in the first category remanufacturing items in second category and third category has such inventory which is returned from the market. This paper provides an optimal solution to control the manufacturing, remanufacturing and disposal rate using sensitivity analysis.

(Singh et al.,2011) studied an inventory model for that inventory which is subjected to continuous deterioration and have a maximum lifetime under two level trade credits period by using convex fractional programming to obtain an optimum solution at reduced inventory cost. (Shah et al.,2014) analysed the problem of handling imperfect product quality which is exposed to deterioration at a constant rate.

They also examined the different challenges that a retailer faces. This paper aims to develop a model based on advanced preservation technology to maximize profit of the retailer using a sensitivity analysis.

The proposed model by which an optimum solution is derived as a new approach of third order linear equation for those items whose demand changes with time and have a constant deterioration rate. How-ever in real life situation there is inventory loss by deterioration, also a great deal of effort has been focused on the modeling of the production planning problem in deterministic environment. The assumption of a constant demand rate may not be always appropriate for consumer goods such as milk, meat, vegetables, radioactive materials, volatile liquids, etc. as inventory has a negative impact on demand due to loss of consumer confidence about the production quality.

Hence in formulating inventory model two factors have been of growing interest, first deterioration of items and second variation in the demand rate with time. Here we are trying to propose inventory model for deteriorating items with exponential demand and a unique optimal cycle time exists to minimize the annual total relevant cost.

The rest of the paper is organized as follows: Section 2 represents the assumptions and notations and section 3 represents problem formulation. Finally, the paper summarizes and concludes in section 4.

2. PROBLEM DEFINITION, ASSUMPTIONS AND NOTATION

2.1 Assumption:

The following assumption are used to formulate the problem.

1. The initial inventory level is zero.
2. Lead time is zero.

3. Demand rate is exponential where $D = Ae^{\alpha t}$ where α> 0, t >0 at time t and it is continuous function of time. Here α is a constant.
4. The deteriorating item is constant.
5. The planning horizon is finite.
6. There is no repair or replacement of the deteriorated items.
7. Items are produced / purchased and added to the inventory and the item is a single product; it does not interact with any other inventory items.
8. The production rate is always greater than or equal to the sum of the demand rate.

2.2 NOTATION:

The following notation are used in our Analysis
1. P-Production rate in units per unit time.
2. Q*-Optimal size of production run.
3. C_p - Production Cost per unit.
4. θ-rate of deterioration.
5. C_d- deterioration cost per unit.
6. C_0-SetupCost/ Ordering Cost.
7. C_h-holding cost per unit/year.
8. T-Cycle time.
9. t_1-Production time.
10. T_C-Total Cost.

3. MATHEMATICAL MODEL

3.1 Development of Mathematical Model

The methodology adopted in this paper involves a number of steps. First, the differential inventory equations for all the periods are developed. Next, these differential equation are solved to formulate the cost model. The details of this methodology are discussed below. Let us consider a two-stage production-inventory cycle $[0,T]$ of cycle time $T(T>0)$ as shown in

Figure 1. It shows inventory level $I(t)$ at time $t(t \geq 0)$ for two stages of the cycle, namely the production stage and the consumption stage. Considering the production time t_1 ($0 \leq t \leq t_1$), the production stage covers the period $[0, t_1]$ and the consumption stage covers the period $[t_1, T]$ or $t_1 \leq t \leq T$. This figure represents break up of time when the demand gets reduced

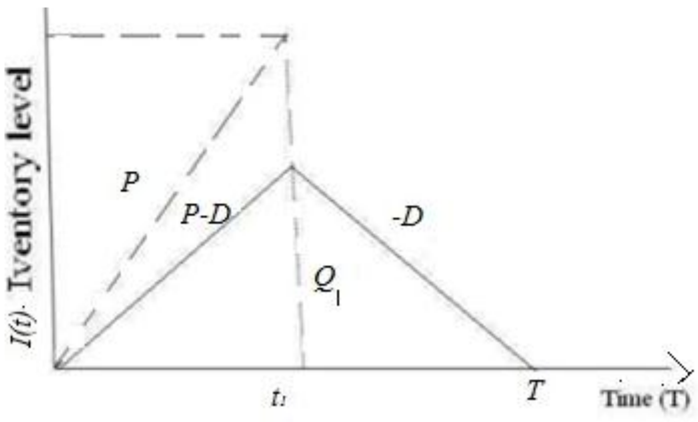

Figure 1: Production-inventory cycle

3.2 Production period [0, t_1]

The production period $[0, t_1]$ is defined as the time taken for the production of desired unit. During this stage, the inventory of items increases due to production at a rate of *P* items per unit of time but decreases due to demand at rate *D* items per unit of time. Here we are interested in exponential demand of $D = Ae^{\alpha t}$, where $\alpha > 0$ is a constant. Note that *D* is a continuous function of time *t*. With boundary conditions $I(0) = 0$, $I(T) = 0$, and $I(t_1) = Q_1$, and a deterioration rate of $\theta(0 < \theta < 1)$, the rate at which inventory changes with respect to time over the production period is given by

$$\frac{dI(t)}{dt} + \theta I(t) = P - Ae^{\alpha t}, \quad \text{for } 0 \leq t \leq t_1. \tag{1}$$

It is also known as inventory differential equation during production period

.

3.3 Consumption period [t_1, T]

During the consumption period **[t_1, T]**, no production occurs and subsequently reduction in the inventory level is due to deterioration items. If such problem arises in a definite time period then this means seasonal type problems and newspaper inventory type problems persist.

For example, after Christmas the demand of cake and cookies get reduced. Also a hike in demand of cold drinks is seen in summer season**.** If there is less demand of items then items start deteriorating as soon as they are produced or after a certain period time.

Thus, the rate at which inventory changes with respect to time over the consumption period is given by

$$\frac{dI(t)}{dt} + \theta I(t) = -Ae^{\alpha t}. \qquad t_1 \leq t \leq T \tag{2}$$

Figure 1 matches to this problem which represents that according to t_1 time demand is decreased (We can say that preservative items like Jam, bread starts detonating at that time t_1) and T represents complete cycle time. The solutions of differential equations given in (1) and (2) respectively are

$$I(t) = \frac{P}{\theta} - \frac{Ae^{\alpha t}}{\theta + \alpha} + \left(\frac{A}{\theta + \alpha} - \frac{P}{\theta}\right) e^{-\theta t}, 0 \leq t \leq t_1 \tag{3}$$

and

$$I(t) = \frac{-Ae^{\alpha t}}{\theta + \alpha} + \left(\frac{Ae^{(\alpha+\theta)T}}{\theta + \alpha}\right) e^{-\theta t}, t_1 \leq t \leq T \quad . \tag{4}$$

Because we know that $I_1(t) = I_2(t)$ at $t = t_1$, then from (3) and (4) we get. Compare that at time t_1

$$\frac{P}{\theta} - \frac{Ae^{\alpha t_1}}{\theta + \alpha} + \left(\frac{A}{\theta + \alpha} - \frac{P}{\theta}\right) e^{-\theta t_1} = \frac{-Ae^{\alpha t_1}}{\theta + \alpha} + \left(\frac{Ae^{(\alpha+\theta)T}}{\theta + \alpha}\right) e^{-\theta t_1} \tag{5}$$

Expanding the exponential functions and noting that second and higher powers of θ and α tend to 0 with increasing power we assume that , we get, $t_1 = AT/P$

3.4 Total Inventory Cost (TIC)

The total inventory cost (TIC) comprises of the ordering cost, holding cost and deteriorating cost, i.e. TIC = Ordering Cost + Holding Cost + Deteriorating Cost.

These costs are evaluated individually as follows:

i. **Ordering Cost per unit time** (O_c)**:** Suppose C_0 is the total set-up or ordering cost, then the ordering cost per unit time for a cycle over period $[0,T]$ is given by $O_c = C_o/T$.

ii. **Holding Cost/unit per unit time** (H_c)**:**

$$H_C=\frac{C_h}{T}\left[\int_0^T I(t)dt\right] \quad (6)$$

$$H_C=\frac{C_h}{T}\left[\int_0^{t_1} I_1(t)dt+\int_{t_1}^{T} I_2(t)dt\right]$$

We know that equation (3)

$$I_1(t)=\frac{P}{\theta}-\frac{Ae^{\alpha t}}{\theta+\alpha}+\left(\frac{A}{\theta+\alpha}-\frac{P}{\theta}\right)e^{-\theta t}, 0\le t\le t_1$$

Taking approximate value of $I_1(t)$, putting $e^{\alpha t}$=1+αt and $e^{\theta t}$=1+θt then we get

$$I_1(t)=Pt-\frac{\alpha At}{\theta+\alpha}-\left(\frac{\theta At}{\theta+\alpha}\right), 0\le t\le t_1$$

$$I_1(t)=[Pt-At], \qquad 0\le t\le t_1 \quad (7)$$

We know that equation 4th

$$I_2(t)=\frac{-Ae^{\alpha t}}{\theta+\alpha}+\left(\frac{Ae^{(\alpha+\theta)T}}{\theta+\alpha}\right)e^{-\theta t}, t_1\le t\le T$$

Taking approximate value of $I_2(t)$, put $e^{\alpha t}$=1+αt and $e^{\theta t}$=1+θt then we get

$$I_2(t)=\frac{-A(1+\alpha t)}{\theta+\alpha}+\frac{A}{\theta+\alpha}+\frac{A(\alpha+\theta)T}{\theta+\alpha} t_1\le t\le T$$

$$I_2(t) = \frac{-\alpha At}{\theta + \alpha} + AT - \frac{A\theta t}{\theta + \alpha} - \theta ATt$$

$$I_2(t) = (AT - At - \theta ATt)t_1 \leq t \leq T \qquad (8)$$

Now

$$\int_0^T I(t)dt = \int_0^{t_1} I_1(t)dt + \int_{t_1}^T I_2(t)dt$$

$$\int_0^T I(t)dt = \int_0^{AT/P} I_1(t)dt + \int_{AT/P}^T I_2(t)dt$$

We will get

$$\int_0^T I(t)dt = \frac{AT^2}{2} - \frac{\theta AT^3}{2} - \frac{A^2T^2}{2P} + \frac{\theta A^3T^3}{2P^2}$$

Now from 6th equation (Holding Cost/unit) becomes

$$H_C = \frac{C_h}{T}\left[\frac{AT^2}{2} - \frac{\theta AT^3}{2} - \frac{A^2T^2}{2P} + \frac{\theta A^3T^3}{2P^2}\right] \qquad (9)$$

iii. **Deteriorating Cost/Unit time Therefore, DC**

$$D_C = \frac{\theta C_d}{T}\left[\int_0^T I(t)dt\right] \qquad (10)$$

We know that

$$\int_0^T I(t)dt = \frac{AT^2}{2} - \frac{\theta AT^3}{2} - \frac{A^2T^2}{2P} + \frac{\theta A^3T^3}{2P^2}$$

Now from 9th equation becomes

$$D_C = \frac{\theta C_d}{T}\left[\frac{AT^2}{2} - \frac{\theta AT^3}{2} - \frac{A^2T^2}{2P} + \frac{\theta A^3T^3}{2P^2}\right]$$

Now

$$T_C = \frac{C_0}{T} + \frac{C_h + \theta C_d}{T}\left(\int_0^T I(t)dt\right)$$

$$T_C = \frac{C_0}{T} + \frac{C_h + \theta C_d}{T}\left[\frac{AT^2}{2} - \frac{\theta AT^3}{2} - \frac{A^2T^2}{2P} + \frac{\theta A^3T^3}{2P^2}\right]$$

We assume

$$C = \frac{C_0}{A(C_h + \theta C_d)}\left(1 - \frac{A}{P}\right)^{-1}$$

$$T_C = (C_h + \theta C_d)\left(1 - \frac{A}{P}\right)\left[\frac{AC}{T} + \frac{AT}{2} - \frac{\theta AT^2}{2} - \frac{\theta A^2T^2}{2P}\right] \qquad (11)$$

For optimum conditions:

$$\frac{dT_C}{dT} = 0, \frac{d^2T_C}{dT^2} > 0$$

Differentiate w.r.t T we get

$$\frac{dT_C}{dT} = -\frac{AC}{T^2} + \frac{A}{2} - \theta AT - \frac{\theta A^2 T}{P} = 0$$

$$\frac{dT_C}{dT} = 0$$

$$\Rightarrow -AC + \frac{AT^2}{2} - \theta AT^2 - \frac{\theta A^2 T^3}{P} = 0$$

$$\Rightarrow -\theta T^3\left(1 + \frac{A}{P}\right) + \frac{T^2}{2} - C = 0$$

$$T^3 - \frac{T^2}{2k} + \frac{C}{2k} = 0$$

We assume

$$k = \theta\left(1 + \frac{A}{P}\right)$$

Assumption for solving above cubic type equation

Let

$$T = \left(x + \frac{1}{6k}\right) \qquad (12)$$

where

$$k = \theta\left(1 + \frac{A}{P}\right)$$

$$\left(x + \frac{1}{6k}\right)^3 - \frac{1}{2k}\left(x + \frac{1}{6k}\right)^2 + \frac{C}{2k} = 0$$

$$x^3 + \frac{1}{216k^3} + \frac{1}{2k}x^2 + \frac{1}{12k^2}x - \frac{x^2}{2k} - \frac{1}{72k^3} - \frac{x}{6k^2} + \frac{c}{2k} = 0$$

$$x^3 - \frac{x}{6k^2} + \frac{c}{2k} - \frac{1}{108k^3} = 0$$

$$u^3 + v^3 = -\frac{c}{2k} + \frac{1}{108k^3}$$

$$u^3 v^3 = \frac{1}{216k^6}$$

$$x^2 - (u^3 + v^3)x + u^3 v^3 = 0$$

$$u^3 = -\frac{c}{4k} + \frac{1}{216k^3} + \sqrt{\frac{C^2}{16K^2} - \frac{C}{6^3 4k^4} - \frac{215}{6^6 k^6}}$$

$$v^3 = -\frac{c}{4k} + \frac{1}{216k^3} - \sqrt{\frac{C^2}{16K^2} - \frac{C}{6^3 4k^4} - \frac{215}{6^6 k^6}}$$

Now from 12th equation becomes

$$T = (u + v + \frac{1}{6k}) \qquad \therefore x = u + v$$

we assumed

$$l = \frac{1}{6k}$$

$$T = (u + v + l) \tag{13}$$

Substitute the value u and v value in equation (13) we will get

Hence

$$T = l + \sqrt[3]{\frac{-3cl}{2} + l^3 + \sqrt{\frac{9C^2 l^2}{4k^2} - 3cl^4 - 215l^6}} + \sqrt[3]{\frac{-3cl}{2} + l^3 - \sqrt{\frac{9C^2 l^2}{4k^2} - 3cl^4 - 215l^6}}$$

Which is the standard inventory model.

Where

$$\frac{1}{l} = 6\theta(1 + \frac{A}{P})$$

$$(C_h + \theta C_d)\frac{A}{C_0}(1 - \frac{A}{P}) = \frac{1}{C}$$

The above standard inventory model gives optimal time. If we run optimal time then we get optimal solution. For the purpose of management of production inventory with reduced cost and maximize profit for deterioting items such as vegetable, milk, meat etc with exponential demand.

3.5 Numerical Example

A manufacturing company plans to use an EOQ Economic order quantity) approach in planning its annual production of 60,000 gears. The set-up and ordering cost add to 4,000 per set-up. The inventory carrying cost per month is established at 2% of the average inventory value. Each gear costs of selling is 250 in the market. Determine Demand, T(Cycle Time/ per unit of product), t_1(production time(at this time demand decreases)/ break time), Q*(optimal size of production run), Setup Cost per unit , Deteriorating cost gear per unit ,and the total inventory costs.

Solution: Given P=60, 000 gears per year, C_o= 4,000 per set, per set cost define by One set if we make one set up for which we use many set. C_h=24%of Rs 250=Rs.60 per year .C=250 per gear, Here one set holding cost is define by C_h. In table below is define per set up holding cost per year and C define one gear cost .D=$Ae^{\alpha t}$ where A=1 and α=1 To find simple solution we take constant equal to 1. D=$Ae^{\alpha t}$ where A=1 and α=1. **EOQ** is also referred to as the optimum lot size.

$$\text{EOQ}(Q^*) = \sqrt{\frac{2DC_0}{C_h}\left(\frac{P}{P-D}\right)}$$

$$T = l + \sqrt[3]{\frac{-3cl}{2} + l^3 + \sqrt{\frac{9C^2l^2}{4k^2} - 3Cl^4 - 215l^6}} + \sqrt[3]{\frac{-3Cl}{2} + l^3 - \sqrt{\frac{9C^2l^2}{4k^2} - 3Cl^4 - 215l^6}}$$

$$t_1 = \frac{AT}{P}$$

ANALYSIS AND RESULTS

Table 1: Production inventory model for deteriorating items with exponential Demand

Deteriorati-on rate θ	**Demand (in gears per unit/per year)**	**Cycle Time(T) per unit of product**	**Production time/ break time t_1**	**Optimal size of production run Q^***	**Setup Cost per unit/per year**	**Holding Cost per unit/per year**	**Deteriorating cost per unit/per year**	**Total invento-ry cost per year**
0.0100	365.10585	17.398745	0.000290	11.548776	229.901642	431.138733	17.964113	679.004517
0.0200	365.050005	8.219807	0.000137	11.547893	486.629456	206.050964	17.170914	709.851318
0.0300	365.031755	5.220623	0.000087	11.547604	766.192078	132.086670	16.510834	914.789551
0.0400	365.02263	3.723144	0.000062	11.547461	1074.360718	95.058304	15.843050	1185.262085
0.0500	365.017155	2.821389	0.000047	11.547373	1417.741333	72.699913	15.145815	1505.587036

0.0600	365.013505	2.216748	0.000037	11.547315	1804.445313	57.656178	14.414044	1876.515503
0.0700	365.01095	1.781920	0.000030	11.547273	2244.769287	46.788715	13.646708	2305.204590
0.0800	365.00876	1.453412	0.000024	11.547241	2752.144287	38.531860	12.843954	2803.520020
0.0900	365.0073	1.195974	0.000020	11.547216	3344.553711	32.016670	12.006252	3388.576660
0.1000	365.00584	0.988449	0.000016	11.547196	4046.742188	26.721893	11.134122	4084.598145

Figure 2: Represents Cycle time as a function of **deteriorating** *cost*

This graph represents according to cycle time gives us deteriotion cost that means any product very easily find the deteriotion cost within one cycle time. From the table 1, a study of rate of deteriorative items with cycle time, setup cost and total cost and it is concluded that when the rate of deteriorative items increases then the setup cost and total cost increases then it is positive relationship between them and the demand, cycle time, deteriorating cost, optimum quantity and holding cost decreases then there is negative relationship between them.

SENSITIVITY ANALYSIS:

The total cost functions are the real solution in which the model parameters are assumed to be static values. It is reasonable to study the sensitivity i.e. the effect of making chances in the model parameters over a given optimum solution. It is important to find the effects on different system performance measures, such as cost function, inventory system, etc. For this purpose, sensitivity analysis of various system parameters for the models of this research are required to observe whether the current solutions remain unchanged, the current solutions become infeasible, etc.

4. DISCUSSION AND CONCLUSION

In this paper, a production inventory model for deteriorating items with exponential time dependence of demand of the form $D = Ae^{\alpha t}$; $A \geq 0$, $\alpha \geq 0$ is considered in which the optimum solutions are derived from the cubic order linear equation as a new approach to the inventory literature or time-dependent demand patterns. This type of demand has a better representation of time-varying market. The objective of this research is to develop a production inventory model for deteriorating items with exponential demand and a unique optimal cycle time exists to minimize the annual total relevant cost. The relevant model is built and solved. Necessary and sufficient conditions for optimal solution are derived. The model proposed in this paper can be extended in several ways.

(i) This model extend to realistic features such as quantity discounts, permissible delay in payments, time value of money, a finite rate of replenishment, inflation, etc.

(ii) In this model demand as a function of quantity as well as quadratic times varying could be considered.

(iii) By this model can be generalized with stochastic market demand.

(iv) In this model we provide some numerical results to illustrate the model.

(v) This model express present research reveal the impact of the above mentioned factors and it in lower cycle time calculation and gives breakeven point for such cases.

(vi) To develop production inventory model is one of the most important function of a business organization group which strictly says that even a small increase in production time of each component may lead to huge losses in business. Thus, the production decisions are very important as they affect different aspects such as demand and supply of a product, inventory management, deterioration rate and deterioration cost all these aspects controlled by this model and gives optimal solution.

(vii) To create availability of human resources and standardized tools along with the equipment's to accomplish the objective of minimum cost, minimum time and maximum output is one of the major challenges that a firm faces in production decision making.

(viii) To evaluate this study will help a business to make an optimum utilization of such deteriorating items to generate more profit at the minimum cost and time.

APPENDIX –A

Solving Procedure- Exponential demand with third order equation.

Consider the cubic equation $ax^3 + bx^2 + cx + d = 0$.

Letting $x = y - (b/3a)$ reduces it to if we substitute in cubic equation we will get

$$y^3 = py + q = 0$$

Assume $y = u + v$

$$u^3 + v^3 + 3uvy = py + q$$

$$u^3 + v^3 = q$$

$$3uv = p$$

$$uv = \frac{p}{3} \rightarrow u^3v^3 = \frac{p^3}{27}$$

There are the roots are quadratic

$$x^2 - (u^3 + v^3)x + u^3v^3 = 0$$

$$x^2 - qx + \frac{p^3}{27} = 0$$

$$u = \sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} - \frac{p^3}{27}}}$$

$$v = \sqrt[3]{\frac{q}{2} - \sqrt{\frac{q^2}{4} - \frac{p^3}{27}}}$$

$$y = u + v$$

$$x = y - \frac{b}{3a}$$

Now find roots

$$x_1 = u + v - \frac{b}{3a}$$

$$x_2 = u\omega + v^3 - \frac{b}{3a}$$

$$x_3 = u\omega^2 + v\omega - \frac{b}{3a}$$

For Example

$$x^3 = 1 \; solution \, are \; x = 1, \omega, \omega^2$$

REFERENCES

Ballou, Ronald H.(2007). *Business logistics/supply chain management, 5/E (With Cd).* Pearson Education India.

Bouras, Abdelghani,(2015). *Optimal Control for Advertised Production Planning in a Three-Level Stock System with Deteriorating Items*: *Case of a Continuous-Review Policy*, *Arabian Journal for Science and Engineering* **40**(9):2829-2840.

Chung, Kun-Jen, and Sui-Fu Tsai., (2001). *Inventory systems for deteriorating items with shortages and a linear trend in demand-taking account of time value*, Computers & Operations Research **28**(9): 915-934.

Dave, Upendra, and L. K. Patel, (T, S i) (1981). *policy inventory model for deteriorating items with time proportional demand*, Journal of the Operational Research Society **32**(2) :137-142.

Mishra, Vinod Kumar, Lal Sahab Singh, and Rakesh Kumar, (2013). *An inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging*, Journal of Industrial Engineering **9**(1): 4.

Pahl, Julia, and Stefan Voß., (2014). I*ntegrating deterioration and lifetime constraints in production and supply chain planning: A survey*, European Journal of Operational Research. **238**(3) : 654-674.

Rastogi, Manoj Kumar (2010). *Production and operation management*, Laxmi Publications, Ltd..

Sarkar, T., S. K. Ghosh, and K. S. Chaudhuri, (2012). *An optimal inventory replenishment policy for a deteriorating item with time-quadratic demand and time-dependent partial backlogging with shortages in all cycles*, Applied Mathematics and Computation. **21**(18) :9147-9155.

Sazvar, Zeinab, Armand Baboli, and Mohammad Reza Akbari Jokar, (2013). *A replenishment policy for perishable products with non-linear holding cost under stochastic supply lead time*, The International Journal of Advanced Manufacturing Technology **64**(5):1087-1098.

Shah, Nita H., and Arpan D. Shah. (2014). *Optimal cycle time and preservation technology investment for deteriorating items with price-sensitive stock-dependent demand under inflation*, *Journal of Physics: Conference Series*. (1): **495**.

Singh, S., Rakesh Dude, and R. Singh, (2011). *Production model with Selling Price dependent demand and Partial Backlogging under inflation*, International Journal of Mathematical Modelling & Computations. **1**(1): 1-7.

Widyadana, Gede Agus, and Hui Ming Wee., (2012).*An economic production quantity model for deteriorating items with preventive maintenance policy and random machine breakdown*, International Journal of Systems Science. **43**(10):1870-1882.

Yadav, Dharmendra, et al., (2015).*Manufacturing inventory model for deteriorating items with maximum lifetime under two-level trade credit financing*, International Journal of Computer Applications. (15) **: 121**.

Yong, Shou-Yang Wang, and Kin Keung Lai., (2010). *An optimal production-inventory model for deteriorating items with multiple-market demand*, European Journal of Operational Research. **203**(3):593-600.